FUNDAÇÃO
GETULIO VARGAS

EPGE
Escola de Pós-Graduação
em Economia

Ensaios Econômicos

Escola de
Pós-Graduação
em Economia
da Fundação
Getulio Vargas

N° 368 ISSN 0104-8910

Corruption And The Choice Of Auction Format

Flavio Marques Menezes, Paulo Klinger Monteiro

Fevereiro de 2000

URL: http://hdl.handle.net/10438/961

Marques Menezes, Flavio
Corruption And The Choice Of Auction Format/
Flavio Marques Menezes, Paulo Klinger Monteiro - Rio de Janeiro
: FGV,EPGE, 2010
(Ensaios Econômicos; 368)

Inclui bibliografia.

CDD-330

Corruption and the Choice of Auction Format

Flavio M. Menezes and Paulo K. Monteiro
Graduate School of Economics
Getulio Vargas Foundation
EPGE/FGV
Praia de Botafogo 190, sala 1125
Botafogo, Rio de Janeiro, RJ, 22253-900
Brazil
Email: flavio.menezes@fgv.br and pklm@fgv.br

February 18, 2000

Abstract

We investigate the outcome of an auction where the auctioneer approaches one of the two existing bidders and offers an opportunity for him to match his opponent's bid in exchange for a fixed bribe. In particular, we examine two types of corruption arrangements. In the first case, the auctioneer approaches the winner to offer the possibility of a reduction in his bid to match the loser's bid in exchange for a bribe. In the second arrangement, the auctioneer approaches the loser and offers him the possibility of matching the winner's bid in exchange for a bribe. Oral auctions are shown to be corruption free under the two arrangements. Corruption, however, affects both bidding behavior and the seller's expected revenue in a first-price auction. In the first case there is an increasing symmetric bidding strategy equilibrium and, therefore, the object always goes to the individual with the highest valuation. As a result, efficiency is not compromised. In the second case there is a symmetric bidding strategy equilibrium involving constant strategies in part of the domain. Therefore, the object does not go necessarily to the individual with the highest valuation. We compute the seller's expected revenue and the expected bribe in both cases.

1 Introduction

Governments typically sell commodities (or procure contracts) using first price sealed-bid auctions, while similar sales by private owners are made through English oral ascending-price auctions. We propose an explanation for this choice when bidders have independent private values and are risk neutral.[1]

A naive commentator could argue that the difference occurs because governments expect to raise more revenue this way, but auction theory establishes that when bidders are risk neutral and under private independent values both types of auction raise the same expected revenue. Allowing bidders to be risk averse does not explain the choice of auction format either, because more revenue is raised in a first price auction, and private sellers would lose by choosing the oral auction. Moreover, when values are affiliated and agents are risk neutral, oral auctions generate more expected revenue than first price auctions. With affiliated values and risk averse bidders, the ranking of auctions according to the expected revenue they generate is ambiguous.

We find an explanation for the choice of auction type in this paper by allowing for corruption between the auctioneer and one of the bidders. The gains from corruption might arise in a first price auction because the auctioneer can sell a bidder information about the bids made by others. A bidder will pay for this information if he values the commodity being sold above the bids made by all others. There are at least two ways that corruption can take place; the first, where the auctioneer invites a bidder to "resubmit" his bid after all the bids are lodged with the auctioneer, or the second, where the auctioneer agrees to inform a bidder of the highest bid just prior to the closing time for bids. We will argue that in both circumstances there are no gains from corruption in an oral auction where bidders have independent private values because it is still a dominant strategy to bid one's true value. In a first-price auction, however, we show that the seller's revenue falls. In

[1] Klemperer (1998) offers an alternative explanation for the possible choice of first-price auctions. With almost common values – a common values setting where each player has a signal about the true valuation of the object (unknown at the time of bidding) and there are small asymmetries between players – first-price auctions generate more revenue than oral auctions. The intuition is that giving a bidder a slightly higher value when he wins an oral auction makes him bid more aggressively and, consequently, his opponents face an increased winner's curse and bid less aggressively. This process continues resulting in a reduced revenue for the seller in an oral auction.

this case, private sellers might choose an oral auction because it is corruption proof. In contrast, government bureaucrats have an incentive to choose a first price auction because it gives them an opportunity to collect rents through bribery.

This paper draws from two different fields: the economic theory of corruption and the theory of auctions. Becker and Stigler (1975) argue that either law enforcement should be privatized or salaries of law enforcers should be increased to reduce the likelihood of corruption, where privatization would "unleash the powerful forces of competition." Rose-Ackerman (1975,1978) argues that while in some circumstances corruption may be efficient (e.g., in the case where a bureaucrat is not well paid and can supplement his salary through taking bribes), there are situations where corruption should be prevented because it results in socially undesirable outcomes (due to informational problems) (e.g., where a driving license is given to a visually impaired person). Rose-Ackerman also points out that markets conditions are crucial in determining the circumstances under which corruption occurs. This argument is also true in our framework.

Krueger (1974), Bhagwati (1982) and Beck and Maher (1986) emphasize the efficiency aspects of corruption, especially in underdeveloped countries. Lui (1985), for instance, examines an equilibrium queuing model where bribery is efficient, while Myrdal (1968), on the other hand, claims that corrupt officials may deliberately cause administrative delays so as to attract bribes. If this is the case, then the efficiency argument is less appealing. Similarly, in our model, corrupt officials might have chosen the auction format so as to maximize their gains, and this can reduce efficiency.[2]

Previous studies of collusive behavior in auctions have focused on collusion between bidders (bidding rings).[3] Laffont and Tirole (1991) examine the design of auctions to favor specific bidders, such as the case of government procurement favoring domestic suppliers. Graham and Marshall (1987) and Malaith and Zemsky(1991) show that in second-price sealed-bid and English

[2] The argument is that private sellers, in contrast, are free to choose the auction house. In this competitive environment, auction houses end up offering only oral auctions because they are corruption proof.

[3] The only exception is Beck and Maher (1986), who compare competitive bidding (in the form of a first price sealed bid auction) and a bribery competition, and find that the expected payoffs are the same under the two allocation mechanisms. In contrast, by allowing corruption between the auctioneer and one of the players, we show that seller's expected revenue and the expected bribe may depend on how the bribery market is organized.

oral auctions, when collusion among players is allowed, cooperative strategies are dominant. The optimal response of the auctioneer is to establish a reserve price that is a function of the coalition's size. Furthermore, they show that the revenue equivalence between second-price and English auctions holds.

McAfee and McMillan (1992) examine collusion among players in first-price auctions where they show that the price paid is the minimum price. Again, sellers can react by increasing the reserve price. Notice that there is no clear cut way of ranking first- and second-priced auctions when collusion is possible, because one can construct examples where the revenue from oral auctions with collusion will be greater than the revenue from a first-priced auction, or vice-versa, depending on the underlying assumptions. Thus, the work on collusion does not explain why governments generally choose first-priced sealed bid auctions when private auctions are typically oral, because both are subject to this form of collusion.

However, this is not the case when there is collusion involving the auctioneer (we refer to this as corruption), because the oral auction is immune to this type of corruption. We should point out that while McAfee and McMillan cite evidence for collusion between bidders for government contracts where "it has been often the case that all bids are identical to the last cent," this could also be evidence of corruption involving the auctioneer. We claim that any evidence based on little dispersion amongst observed bids can also be construed as evidence of corruption.

In this paper we investigate the outcome of an auction where the auctioneer approaches one of the two existing bidders and offers an opportunity for him to match his opponent's bid in exchange for a bribe. In particular, we examine two cases of corruption. In the first case, the auctioneer approaches the winner of the auction to offer a reduction in his bid to match the loser's bid in exchange for a bribe. In the second case the auctioneer approaches the loser and offers him the possibility to match the winner's bid in exchange for a bribe. Therefore, if corruption does occur, the published bids will be identical. (In practice, the observed winner's bid might be a few cents above the observed loser's bid).

Oral auctions are corruption free in both cases. Corruption, however, affects both bidding behavior and the seller's expected revenue in a first-price auction. In the first case there is an increasing symmetric equilibrium bidding strategy and, therefore, the object always goes to the individual with the highest valuation. Thus, efficiency is not compromised. In the second case there is a symmetric bidding strategy equilibrium involving constant strategies in part of the domain. In this case the object does not go necessarily to the individual with the highest valuation. We also compute the seller's expected revenue and the expected bribe in both cases.

2 The Model

There are two risk-neutral bidders who are competing for an object to be auctioned off. According to the independent private values assumption, Bidder $i \in I$ knows his own value (v_i) for the object but only knows the distribution $F(v_j)$, $\forall j \neq i$, of the other bidder's value. It is assumed that values are independently drawn from the Uniform $[0, 1]$ distribution.[4]

The auctioneer is corrupt. We consider two possible types of corrupt practices. In the first case, the auctioneer approaches the winner and offers him to reduce his bid to the second highest bid in exchange for a bribe B. If this bidder agrees to pay the bribe, then he wins the object paying what is effectively the second highest bid (plus the bribe to the auctioneer). If this player does not agree to pay the bribe, then he still wins the object and pays accordingly to the auction rules — e.g., his bid when a first price auction is used. In the second case, the auctioneer approaches the loser and offers him to increase his bid to match the highest bid in exchange for a bribe B. If this player agrees, he wins the auction and pays this highest bid. Otherwise, this player loses the auction. Note that the bribery arrangement is common knowledge. Moreover, both bribery arrangements are consistent with the evidence of little dispersion amongst observed bids as discussed in the previous section because under corruption the two observed bids will be identical.

In this paper the size of the bribe, B, is fixed. For example, B might have been determined by a social convention or norm. Although a more complete model would have B as endogenous, perhaps as a fraction of the winning bid, the analysis of fixed B already yields several interesting results. We also ignore the principal-agent relationship between the seller and the auctioneer. As mentioned earlier, our example is that of a government official who has a choice between an oral auction and a first price auction to sell a government-

[4]In the appendix we consider a setup with n bidders and a general distribution of values when the auctioneer approaches the winner. To obtain close form solutions for the equilibrium bidding strategies and to compute the seller's expected revenue and the expected bribe we limit ourselves to the two-bidder uniformly-distributed values case.

owned good. Next we argue that an oral auction is corruption proof.[5] In the next section we investigate under which conditions corruption will occur in a first price auction.

Proposition 1 *Under the two alternative bribery arrangements, oral auctions are corruption free.*

Proof. We will now demonstrate that it is a dominant strategy to bid one's true valuation and that there is no corruption in equilibrium. We will consider the two bribery arrangements separately. First, assume that the auctioneer approaches the winner of the auction. In this case the argument is completely analogous to the standard case without corruption. Assume that Bidder 2 bids an arbitrary number b_2 and Bidder 1 bids $b_1 > v_1$. Bidder 1 does not gain anything by bidding more than his value but may as well lose if $b_1 > b_2 > v_1$. Similarly, suppose Bidder 1 bids $b_1 < v_1$. Bidder 1 does not gain by bidding less than his value but may as well lose the auction (and consequently the object because the auctioneer approaches only the winner) when it should have won, that is, when $v_1 > b_2 > b_1$. Note that there is never corruption because the winner always pays the second highest bid (without the need to pay a bribe).

Now suppose that the auctioneer approaches the loser of the auction. Assume that Bidder 2 bids an arbitrary number b_2 and Bidder 1 bids $b_1 > v_1$. As above, Bidder 1 does not gain anything by bidding more than his value but may as well lose if $b_1 > b_2 > v_1$ and Bidder 2 decides not to pay the bribe. Now suppose Bidder 1 bids $b_1 < v_1$. When $b_1 > b_2$ Bidder 2 is offered the opportunity to match 1's bid and win the auction. In this case, by increasing his bid (up to v_1), Player 1 would increase his expected profits. If $v_1 > b_2 > b_1$, then Player 1 is offered to match 2's bid and pay the bribe. By increasing his bid, Player 1 would increase his expected profits by not having to pay the bribe. As in the previous case, there is never corruption because the winner always pays the second highest bid (without the need to pay a bribe). ■

[5] In this paper we do not distinguish between second-price sealed-bid and English auctions; they are strategically equivalent in an independent-private values model.

3 Corruption in First-Price Auctions

In this section we investigate the effects of corruption on bidding behavior and the seller's expected revenue in a first price auction. We also compute the probability of corruption, investigate the efficiency of first price auctions under corruption and compute the seller's expected revenue. We examine the two bribery arrangements separately.

3.1 The winner is approached by the auctioneer.

We consider the case when the auctioneer approaches the winner and offers him the possibility to match the second highest bid in exchange for a bribe. In the next proposition we characterize a symmetric equilibrium bidding strategies.

Proposition 2

$$b(v) = \begin{cases} \frac{v}{2} & \text{if } \; 0 \leq v \leq 2B \\ \frac{v}{2} + B - \frac{\sqrt{4Bv - v^2}}{2} & \text{if } \; 2B \leq v \leq 1 \end{cases}$$

is an equilibrium if $B\left(2+\sqrt{2}\right) \geq 1$, *that is, if* $B \geq 0.292\,89$ [6]

Proof. If Bidder 2 plays $b(\cdot)$ let us find Bidder 1 best response. If Bidder 1 bids $x \geq 0$ his expected utility is

$$h(x) = E\left[(v - b(y) - B)\,\chi_{b(y)<x-B} + (v - x)\,\chi_{x>b(y)>x-B}\right]. \qquad (1)$$

Let us consider first $x \leq B$. Then $x - B \leq 0$ and therefore $h(x) = (v - x)\Pr(x > b(y)) = (v - x)\, b^{-1}(x)$. Since $x \leq B$, $b^{-1}(x) = 2x$. Thus $h(x) = (v - x)\, 2x$. This function is maximized at $x = \frac{v}{2}$ if $v \leq 2B$ and at $x = B$ if $v > 2B$. Let us now consider $B > x - B > 0$.

$$\begin{aligned} h(x) &= E\left[\left(v - \frac{y}{2} - B\right)\chi_{y<2(x-B)}\right] + (v - x)\Pr(x > b(y) > x - B) = \\ &\int_0^{2(x-B)} \left(v - \frac{y}{2} - B\right) dy + (v - x)\left(b^{-1}(x) - 2(x - B)\right). \end{aligned}$$

[6]The implicit assumption is that $B \leq .5$ otherwise corruption will never occur and we will have the standard equilibrium bidding strategy $b(v) = \dfrac{v}{2}$.

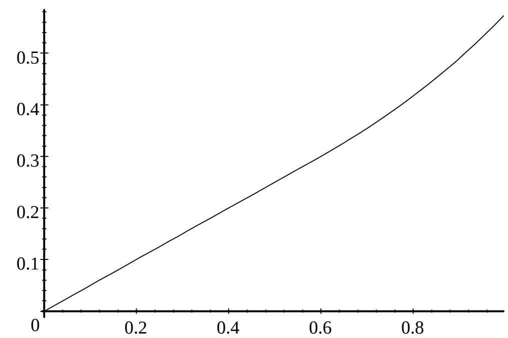


Figure 1: Graph of $b(\cdot)$ ($B = .4$)

The derivative of h is

$$h'(x) = 2(v-x) - \left(b^{-1}(x) - 2(x-B)\right) + (v-x)\left(\left(b^{-1}\right)'(x) - 2\right) =$$

$$-\left(b^{-1}(x) - 2(x-B)\right) + (v-x)\left(b^{-1}\right)'(x) = -\left(b^{-1}(x) - 2(x-B)\right) + \frac{(v-x)}{b'\left(b^{-1}(x)\right)}.$$

We now note that b satisfy the differential equation

$$b'(v) = \frac{v - b(v)}{v + 2B - 2b(v)}.$$

Thus $b'\left(b^{-1}(x)\right) = \frac{b^{-1}(x)-x}{b^{-1}(x)+2B-2x}$. Therefore

$$h'(x) = -\frac{b^{-1}(x) - x}{b'\left(b^{-1}(x)\right)} + \frac{(v-x)}{b'\left(b^{-1}(x)\right)} = \frac{v - b^{-1}(x)}{b'\left(b^{-1}(x)\right)}.$$

Thus h is maximized at $x = b(v)$ in the range $B < b(v) < 2B$. Since $b(v) = 2B$ if and only if $v = \left(2+\sqrt{2}\right)B$ we are done. ■

Given that $B > .29$ is a strong restriction on the bribe B, we proceed to show how to find equilibria for any B. It is an iterative procedure. Define

$$\lambda(v) = \frac{v}{2} + B - \frac{\sqrt{4Bv - v^2}}{2}, 2B \leq v \leq B\left(2+\sqrt{2}\right) \tag{2}$$

and consider the bidding function:

$$b(v) = \begin{cases} \frac{v}{2} & \text{if} \quad 0 \leq v \leq 2B \\ \lambda(v) & \text{if} \quad 2B \leq v \leq B\left(2+\sqrt{2}\right) \\ \psi(v) & \text{if} \quad B\left(2+\sqrt{2}\right) \leq v. \end{cases}$$

We suppose $\psi(\cdot)$ increasing and $\psi\left(B\left(2+\sqrt{2}\right)\right)=2B$. Thus $b(\cdot)$ is increasing and continuous with range $b([0,1])=[0,2B]\cup\left[\psi\left(B\left(2+\sqrt{2}\right)\right),\psi(1)\right]=[0,\psi(1)]$. If Bidder 2 bids accordingly to $b(\cdot)$ then if Bidder 1 has valuation v and bids $x\geq 0$ his expected utility is

$$\begin{aligned} h(x) &= \int_0^{b^{-1}(x-B)} (v-b(y)-B)\,dy + (v-x)\Pr(x>b(y)>x-B) = \\ & \int_0^{b^{-1}(x-B)} (v-b(y)-B)\,dy + (v-x)\left(b^{-1}(x)-b^{-1}(x-B)\right) \end{aligned} \tag{3}$$

The derivative of h is given by

$$h'(x)=\left(b^{-1}\right)'(x-B)(v-x)-\left(b^{-1}(x)-b^{-1}(x-B)\right)+(v-x)\left(\left(b^{-1}\right)'(x)-\left(b^{-1}\right)'(x-B)\right)= \tag{4}$$

$$\begin{aligned} &-\left(b^{-1}(x)-b^{-1}(x-B)\right)+(v-x)\left(\left(b^{-1}\right)'(x)\right)= \\ &-\left(b^{-1}(x)-b^{-1}(x-B)\right)+\frac{(v-x)}{b'\left(b^{-1}(x)\right)}. \end{aligned}$$

In equilibrium $x=b(v)$ and $h'(b(v))=0$. Thus

$$-\left(v-b^{-1}(b(v)-B)\right)+\frac{(v-b(v))}{b'(v)}=0.$$

Or

$$b'(v)=\frac{v-b(v)}{v-b^{-1}(b(v)-B)}. \tag{5}$$

The nature of the inductive construction is as follows. We begin with $b(v)=\frac{v}{2}, 0\leq v\leq 2B$. The differential equation above makes sense if v is such that $b(v)-B\in[0,B]$ which is the range of $b|_{[0,2B]}$. If we define $\lambda(v)=b|_{[2B,\omega]}$ we have that

$$\lambda'(v)=\frac{v-\lambda(v)}{v-b^{-1}(\lambda(v)-B)}=\frac{v-\lambda(v)}{v-2(\lambda(v)-B)}$$

with initial condition $\lambda(2B)=B$. This equation can be explicitly solved and is given by (2). Thus we found $b|_{\left[0,B\left(2+\sqrt{2}\right)\right]}$. To find ψ we consider the initial condition $\psi\left(B\left(2+\sqrt{2}\right)\right)=2B$ and

$$\psi'(v)=\frac{v-\psi(v)}{v-b^{-1}(\psi(v)-B)}=\frac{v-\psi(v)}{v-\lambda^{-1}(\psi(v)-B)}.$$

There exists an $\omega > B\left(2+\sqrt{2}\right)$ such that $\psi(\omega)=3B$. If $\omega \geq 1$ we are done. If not, we follow the inductive approach outlined above.

Note that the equilibrium bidding strategies described above are increasing in the individuals' values. Therefore, the winner of the auction is the individual with the highest valuation. Given that the auctioneer only approaches the winner of the auction, we can conclude that the object (even with corruption) always goes to the individual with the highest valuation.

Corollary 1 *In the equilibrium characterized above first price auctions are efficient.*

The above corollary and the recognition that the individual with the lowest valuation has zero expected profits allow us to conclude that the Revenue Equivalence Theorem holds in this setting. Therefore, the winner's expected total payment in this scenario is identical to his expected payment under any mechanism that allocates the good to the highest bidder – such as the standard first price auction. With two bidders with valuations uniformly distributed on the interval [0,1], the winner's total expected payment is equal to 1/3. In the setting above, this payment is equal to expected bribe plus the payment to the seller. Next we compute the probability of corruption occurring.

Proposition 3 *For $.292 \leq B \leq .5$, the probability of corruption is equal to $1-2B^2$.*

Proof. Suppose Player 1 has a higher valuation. That is, $v_1 > v_2$. Corruption will occur whenever 1's value and Player 2's bid is such that it is feasible for Player 1 to pay the bribe plus the revised bid, that is, whenever $b(v_2)+B < v_1$.
Define $v^1=\max\{v_1,v_2\}$ and $v^2=\min\{v_1,v_2\}$. We can now write the probability of corruption as

$$\Pr[\text{Corruption}]=\Pr[b(v^2)+B < v^1]$$

That is

$$\Pr[\text{Corruption}] = 2\Pr[v_1 > v_2, b(v_2) + B < v_1]$$

Using $b(v)$ we can write

$$\Pr[v_1 > v_2, b(v_2) + B < v_1] = \Pr[v_2 \leq 2B, \frac{v_2}{2} + B < v_1] +$$

$$\Pr[v_1 > v_2 > 2B, b(v_2) + B < v_1].$$

Note that

$$\Pr[v_2 \leq 2B, \frac{v_2}{2} + B < v_1] =$$

$$\Pr[v_2 \leq 2B, v_1 \geq 2B] + \Pr[v_2 \leq 2B, \frac{v_2}{2} + B < v_1, v_1 < 2B]$$

That is

$$\Pr[v_2 \leq 2B, \frac{v_2}{2} + B < v_1] = 2B - 3B^2$$

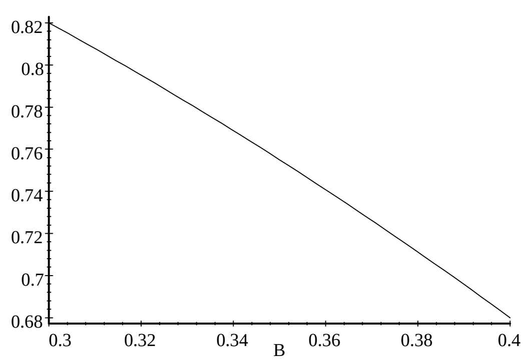
We can check that $b\left(v_2\right) + B < v_2$ for every $v_2 > 2B$. Thus

$$\Pr[v_1 > v_2 > 2B, b(v_2) + B < v_1] = \Pr[v_1 > v_2 > 2B] = \frac{1}{2} - 2B + 2B^2.$$

Therefore,

$$\Pr[\text{Corruption}] = \left(2B - 3B^2 + \frac{1}{2} - 2B + 2B^2\right)2 = 1 - 2B^2.$$

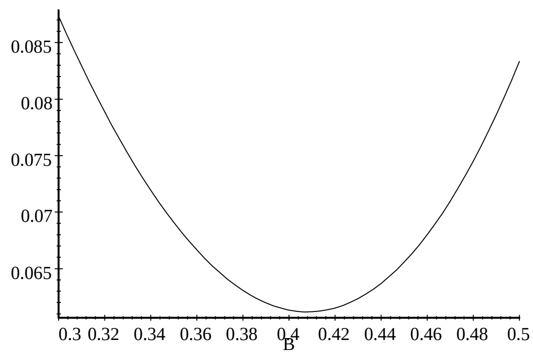
■



Probability of Corruption

In this case, the probability of corruption is quite high. The next result follows from the Revenue Equivalence Theorem given that the expected bribe is simply the bribe times the probability of corruption.

Corollary 2 *For* $.292 \leq B \leq .5$*, the seller's expected revenue is equal to* $\frac{1}{3} - B \times (1 - 2B^2)$.



The seller's expected revenue

That is, when the auctioneer approaches the winner, the seller's expected revenue is reduced quite dramatically for sufficiently large values of B. This demonstrates the potential devastating effects on the government's revenue in very corrupt economies.

Finally, note that changing the way the auctioneer is compensated, say from a fixed salary to a fixed percentage of the sales revenue as commission, may not eliminate or even reduce corruption. Corruption can only be eliminated if the commission is sufficiently large so that the expected commission is larger than the expected bribery. This finding contrasts with that of the classical economic theory of corruption — e.g., Becker and Stigler (1974) – but it is consistent with recent findings (Mookherjee and Png, 1995) that to wipe out corruption a large discrete jump in the official's compensation is required.

3.2 The loser is approached by the auctioneer

We now assume that the auctioneer approaches the bidder who lost the auction to offer him the possibility of revising his bid to match the highest bid and win the auction in exchange for a bribe. In the next proposition we characterize a symmetric equilibrium bidding strategy of this auction game.

Proposition 4

$$b(v) = \begin{cases} \frac{v}{2} & \text{if} \quad v \leq 2B \\ B & \text{if} \quad 2B \leq v \leq 1 \end{cases}$$

is a symmetric equilibrium if $3B \geq 1$.

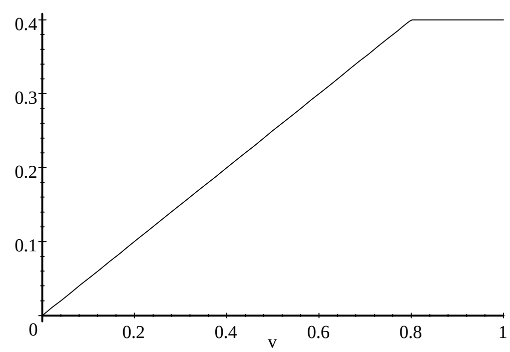


Figure 2: Graph of $b(v)$ $(B = .4)$

Proof. If Bidder 2 bids according to $b(\cdot)$ then if Bidder 1 has a valuation v and bids $x \geq 0$ his expected utility — if there is no possibility of a tie — is equal to

$$h(x) = (v - x) \Pr(x > b(y), x + B > y) + \int_{x<b(y)} (v - b(y) - B)^{+} dy. \quad (6)$$

For this equilibrium we have to consider the possibility of a tie. Here we suppose that in the case of a tie that there is a 50 percent of each bidder being chosen to win the object paying the bribe B and his bid B. For a bid $x \neq B$ Bidder 1's expected utility is $h(x)$ as above. If $x = B$ we redefine h as:

$$h(B) = (v - B) 2B + \frac{v - 2B}{2}.$$

Note also that

$$h(B) = h\left(B^{-}\right) + \frac{v - 2B}{2}.$$

Suppose first that Bidder 1 bids $x > B$. Then $x > b(y)$ is always true. So $h(x) = (v - x)(x + B)$. Since $h'(x) = v - 2x - B \leq 3B - 2x - B \leq 2(B - x) < 0$ the expected utility is never maximized with $x > B$. Since $h(B+) < h(B)$ we have that $\max\{h(x), B \leq x\} = h(B)$.Suppose now that $0 \leq x < B$. Then

$$h(x) = (v - x)\, 2x + \int_{2x}^{1} (v - b(y) - B)^{+}\, dy. \tag{7}$$

The derivative of h is

$$h'(x) = 2v - 4x - 2(v - x - B)^{+} = \left\{ \begin{array}{lll} 2(B - x) & \text{if} & v - B \geq x \\ 4\left(\frac{v}{2} - x\right) & \text{if} & v - B < x. \end{array} \right. .$$

Consider first $v \geq 2B$. Then $h'(x) = 2v - 4x - 2(v - x - B) = -2x + 2B > 0$. Since $h(B-) \leq h(B) = h(B-) + \frac{v-2B}{2}$ the maximum of h will be at $x = B$. I.e. $\max\{h(x); x \leq B\} = h(B)$ if $v \geq 2B$. Consider now $v < 2B$. If $x \leq v - B$, $h'(x) = 2(B - x) > 0$. Thus to maximize h, $x > v - B$. Since $x = \frac{v}{2}$ is such that $v - x - B \leq 0$ we have that h is maximized for $0 \leq x < B$ with $x = \frac{v}{2}$. Thus $\max\{h(x); x \leq B\} = \max\left\{h\left(\frac{v}{2}\right), h(B)\right\}$ if $v < 2B$. It remains to compare $h\left(\frac{v}{2}\right) = \frac{v^2}{2}$ with $h(B) = (v - B)\, 2B + \frac{v-2B}{2}$. Solving the inequality

$$\frac{v^2}{2} \geq (v - B)\, 2B + \frac{v - 2B}{2}.$$

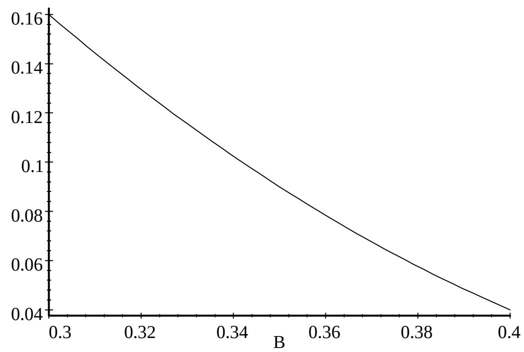
we obtain $v \leq 2B$. This proves that $b(\cdot)$ is an equilibrium. ■

Proposition 5 *The object does not go to the bidder with the highest valuation with probability* $\frac{(1-2B)^2}{2}$.

Proof. Let us suppose that $v > \omega$. The object will go to the bidder with the lowest valuation with probability $\frac{1}{2}$ if and only if $\omega \geq 2B$. Thus the probability that the auction is inefficient is $\frac{(1-2B)^2}{2}$. ■

As corruption occurs only if both players have valuation above $2B$, the next result follows immediately.

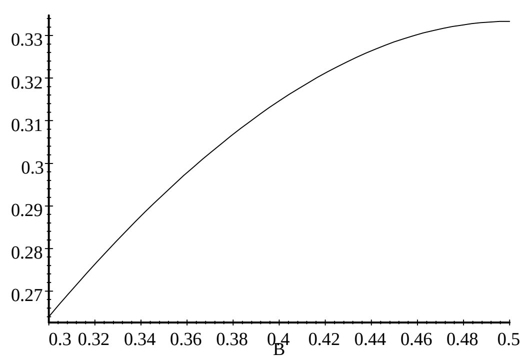
Proposition 6 *The probability of corruption is equal to* $(1-2B)^2$.



Probability of Corruption

The seller's revenue is equal to B if at least one of the players has a value greater than $2B$ – the probability of this event is $\left(\left(1-(2B)^2\right)\right.$. If the two players have values below $2B$ the seller's revenue is equal to the maximum of $\{\frac{v_1}{2}, \frac{v_2}{2}\}$. By taking the expected value we obtain the seller's expected revenue as established in the next proposition.

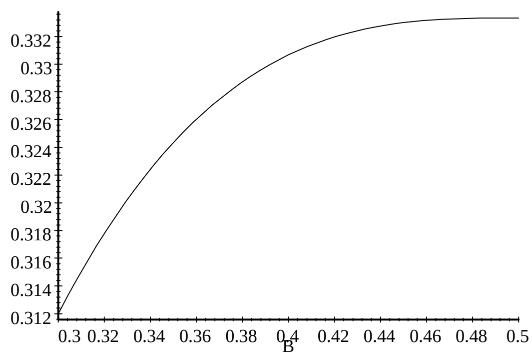
Proposition 7 *The expected bribe is equal to* $B(1-2B)^2$ *and the seller's expected revenue is equal to* $\frac{8}{3}B^3 + B(1-4B^2)$.



Seller's expected revenue

The seller's expected revenue increases with B and approaches the standard revenue when B approaches .5. Moreover, for any B, the seller's expected revenue is larger under this scenario than under the arrangement where the auctioneer approaches the winner of the auction. This can be explained as follows. First, the probability of corruption is substantially larger under the approach-the-winner scenario than under the approach-the-loser scenario. Second, the final bid – that is, the government's revenue – is the second highest bid in the former case under corruption and the highest bid in the latter case. These two effects more than dominate the fact that bidders bid more aggressively under the former scenario than under the latter one.

In the case considered here, as the expected bribe approaches zero as B approaches .5, the winner's expected payment approaches the standard value. Recall that the winner's expected payment in the case where the auctioneer approaches the winner of the auction is equal to 1/3. Consequently, the former scenario generates more surplus than the scenario analyzed in this subsection.



The winner's expected payment

4 Conclusion

In this paper we analyzed the effects of corruption on auctions under the independent private values setting with risk neutral bidders. We examined two distinct arrangements where corruption might occur that are consistent with the evidence of little difference between observed bids. Under these two arrangements, oral auctions are shown to be corruption free as it is still a dominant strategy for a bidder to bid one's true valuation – thus rendering corruption ineffective.

However, bidding behavior in a first price auction is affected by corruption. When the auctioneer approaches the winner of the auction to offer him the possibility of reducing his bid to match his opponent's bid in exchange for a bribe, the revenue equivalence theorem still holds. In the case of a large fixed bribe, the probability of corruption is high and, consequently, the seller's expected revenue is low (as the winning bidder reduces his bid).

When the auctioneer approaches the winner of the auction to offer him the possibility of matching his opponent's high bid (winning the object) in exchange for a bribe, the revenue equivalence theorem fails because now a bidder may want to lose the auction. As a result, the object may go to the individual who values it the least.

These results are only suggestive of how the existence of corruption might affect both the seller's expected revenue and the efficiency of an auction. This seems to be a research line worth pursuing as developing countries, those more naturally prone to corruption, are increasingly using auctions to allocate goods and services and to privatize government owned assets,

References

[1] Beck, P. J. and M. W. Maher, 1986, "A Comparison of Bribery and Bidding in Thin Markets," **Economics Letters** 20, 1-5.

[2] Becker, G. and G. Stigler, 1974, "Law Enforcement, Malfeasance and Compensation of Enforcers," **Journal of Legal Studies 3(1)**, 1-18.

[3] Bhagwati, J., 1982, "Directly Unproductive, Profit-Seeking (DUP) Activities, **Journal of Political Economy 90**, 988-1002.

[4] Graham, D. and R. Marshall, 1987, "Collusive Bidder Behavior at Single-Object Second-Price and English Auctions," **Journal of Political Economy 95(6)**, 1217-1239.

[5] Klemperer, P., 1998, "Auctions with Almost Common Values: The 'Wallet Game' and Its Applications," **European Economic Review 42**, 757-769.

[6] Krueger, A. O., 1974, "The Political Economy of the Rent-Seeking Society," **American Economic Review 64**, 291-303.

[7] Laffont, J. J. and J. Tirole, 1991, "Auction Design and Favoritism," **International Journal of Industrial Organization 9(1)**, 9-42.

[8] Lui, F. T., 1985, "An Equilibrium Queuing Model of Bribery," **Journal of Political Economy 93(4)**, 760-781.

[9] Malaith, G. and P. Zemsky, 1991, "Collusion in Second Price Auctions with Heterogeneous Bidders," **Games and Economic Behavior 3**, 467-486.

[10] McAfee, P. and J. McMillan, 1992, "Bidding Rings," **American Economic Review 82(3)**, 579-599.

[11] Mookherjee, D. and I. P. L. Png, 1995, "Corruptible Law Enforcers: How Should They be Compensated?" **The Economic Journal 105**, 145-159.

[12] Myrdal, G., 1968, **Asian Drama: An Inquiry into the Poverty of Nations**. New York: Pantheon.

[13] Rose-Ackerman, S., 1975, "The Economics of Corruption," **Journal of Public Economics 4**, 187-203.

[14] Rose-Ackerman, S., 1978, **Corruption: A Study in Political Economy**. Academic Press, New York.

A The winner is approached by the auctioneer: The general case

Let us find the equilibrium when there are n Bidders. We are looking for a symmetric increasing equilibrium $b(\cdot)$. Define $Y = \max\{v_j; j \geq 2\}$. If bidders $j \geq 2$ bids accordingly to $b(\cdot)$ and Bidder 1 bids $x \geq 0$ his expected utility is

$$h(x) = (v - x) \Pr(x > b(Y) > x - B) + \int_{b(y) < x - B} (v - b(y) - B)^+ f_Y(y)\, dy.$$

We consider first $0 \leq x < B$. Then $h(x) = (v - x) F_Y(b^{-1}(x))$. This is the usual(i.e. without corruption) expected utility and gives

$$b_1(v) = v - \frac{\int_0^v F(y)^{n-1} dy}{F(v)^{n-1}}. \tag{8}$$

This solution is valid for $0 \leq v \leq v_1$, where v_1 is defined by

$$v_1 - B = \frac{\int_0^{v_1} F(y)^{n-1} \, dy}{F(v_B)^{n-1}}.$$

In the uniform distribution case $v_1 = \frac{n}{n-1} B$. We consider now $B \leq x \leq 2B$. Then

$$h(x) = (v - x) \left(F_Y \left(b^{-1}(x) \right) - F_Y \left(b_1^{-1}(x - B) \right) \right) + \int_0^{b_1^{-1}(x-B)} (v - b_1(y) - B)^+ f_Y(y) \, dy.$$

The first order condition is $h'(x) =$

$$-\left(F_Y \left(b^{-1}(x) \right) - F_Y \left(b^{-1}(x - B) \right) \right) +$$

$$(v - x) \left(f_Y \left(b^{-1}(x) \right) \left(b^{-1} \right)'(x) - f_Y \left(b^{-1}(x - B) \right) \left(b^{-1} \right)'(x - B) \right) +$$

$$\left(b^{-1} \right)'(x - B)(v - x)^+ f_Y \left(b^{-1}(x - B) \right) =$$

$$-\left(F_Y \left(b^{-1}(x) \right) - F_Y \left(b^{-1}(x - B) \right) \right) + (v - x) f_Y \left(b^{-1}(x) \right) \left(b^{-1} \right)'(x) = 0.$$

In equilibrium $x = b(v) < v$ thus

$$-\left(F_Y(v) - F_Y \left(b_1^{-1}(b(v) - B) \right) \right) + \frac{(v - b(v)) f_Y(v)}{b'(v)} = 0.$$

Since $F_Y(z) = F(z)^{n-1}$ and $f_Y(z) = (n-1) f(z) F(z)^{n-2}$ we obtain the differential equation

$$\begin{cases} b'(v) = \frac{(v - b(v))(n-1) f(v) F(v)^{n-2}}{F(v)^{n-1} - F\left(b_1^{-1}(b(v) - B)\right)^{n-1}}, \\ v \geq v_1. \end{cases}$$

with initial condition $b(v_1) = B$. The function, $b_2(\cdot)$, solution of the differential equation above may be defined for all $v \geq v_1$. However we have to restrict its domain to $v \geq v_1$ such that $b_2(v) \leq 2B$ since b_1 has range $[0, B]$. Defined $(b_j(\cdot), v_j)_{j<m}$ we define v_m by $b_{m-1}(v_m) = (m-1)B$ and $b_m(\cdot)$ as solution of the differential equation

$$b'(v) = \frac{(v - b(v))(n-1) f(v) F(v)^{n-2}}{F(v)^{n-1} - F\left(b_{m-1}^{-1}(b(v) - B)\right)^{n-1}}.$$

A.1 The uniform distribution with two bidders case.

To make more clear the steps to find $b(\cdot)$ we particularize to the uniform distribution with two bidders. In this case the function $b_1(v) = \frac{v}{2}$. Thus $v_1 = 2B$. If $2B \geq 1$ we are done. So we suppose $2B < 1$. We now have to solve the differential equation

$$b'(v) = \frac{(v - b(v))}{v - 2(b(v) - B)}, v \geq 2B.$$

The solution of this equation is

$$b_2(v) = \frac{v - \sqrt{4Bv - v^2}}{2} + B.$$

If $b_2(1) \leq 2B$, we are done. This is true if and only if $B \geq \frac{1}{2+\sqrt{2}} = .292\,89$.